Joseph Gardi
Differential Geometry
Notes
Monday, October 10th 2019

Big Ideas

1 Differential geometry is concerend with those properties of surface which depend on their behavior in a neighborhood of a point

2 Accoding to our definition, each point p of a regular surface belongs to a coordinate neighborhood

3 The point of such a neighborhood are characterized by their coodinates and we should be able to define the local properties of interest in terms of these coordinates

A **diffeomorphism** is a differentiable function with a differentiable inverse.

Change of parameters Let p be a point of a regular surface S and let $x : U \subset R^2 \to S, y : V \subset R^2 \to S$ be 2 paremterizations of S such that $p \in x(U) \bigcap y(V) = W$. Then the change of coodinates $h = x^{-1} \circ y : y^{-1}(W) \to x^{-1}(W)$ is a diffeomorphism.

To check that a function is differentiable check that the differential is invertable.

****

Ladies and gentlemen! The times has come to give the definition of **MANIFOLD**:
An abstract surface (differentiable manifold of dimension 2) is a set S together with a family of one-to-one maps $x_\alpha : U_\alpha \to S$ of open sets $U_\alpha \subset R^2$ into S such that,

1 $\bigcup_\alpha x_\alpha(U_\alpha) = S$. That is, the union over the images of the maps in our family of maps is the set S. So S is covered collectivley by our family of maps.

2 For each pair α, β with $x_\alpha(U_\alpha) \bigcap x_\beta(U_\beta) = W = \emptyset$, we have that $x_\alpha^{-1}, x_\beta^{-1}(W)$ are open sets in R^2, and $x_\beta^{-1}(W)$ are open sets in R^2, and $xx_\beta^{-1} \circ x_\alpha, x_\alpha^{-1} \circ x_\beta$ are differentaible maps

The pair (U_α, x_α) is called a differentiable structure on the manifold.