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Differential Geometry
Notes
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Statical manifold
Gaussians form a manifold. You can use differential geometry to crerate a distance function between gaussians. Euclidean distance doesn't work for this.
3 approaches to big data problems
1) Supervised learning: Assume a parametric model and then find parameters that minimize some loss function that measures the error rate on some labeled data set.
2) Probabalistic aprooach: Maximize likelihood of the data
3) Geometric method: Geometric methods let us use visual intuition. For the linear regression problem want to make $y \approx X\theta$ where y is the correct outputs vector, X is an input matrix and θ is the vector with the parameters. $X\theta$ is some linear combination of the columns of X. Suppose that X is n by m. The possible outputs for $X\theta$ is $B = span(\text{columns of } X)$. That is an m dimensional subspace of R^n with the columns of X as it's basis. But y lives in R^n. So we have to find the point in B closest to y. You can do that by projecting y onto B.
Look at the rate of change of The frame $\{x_u, x_v, N\}$ to detect how the manifold is curved.
Christoffel symbols
$x_{uu} = \Gamma_1^{11} x_u + \Gamma_2^{11} x_v + L_1 N$
$x_{uv} = \Gamma_1^{12} x_u + \Gamma_2^{12} x_v + L_2 N$
$x_{vu} = \Gamma_1^{21} x_u + \Gamma_2^{21} x_v + L_2 N$
$x_{vv} = \Gamma_1^{22} x_u + \Gamma_2^{22} x_v + L_2 N$
$N_u = a_{11} x_u + a_{21} x_v$
$N_v = a_{12} x_u + a_{22} x_v$
Then $e = L_1, f = L_2, g = L_3$.
How to find christoffel symbols
Take inner product of both sides with x_u and x_v for the first 4 equations for the christoffel symbols. This gives you a system of equations.

$$< x_{uu}, x_u > = \Gamma_1^{11} < x_u, x_u > + \Gamma_2^{11} < x_v, x_u >$$
$$< x_{uu}, x_v > = \Gamma_1^{11} < x_u, x_v > + \Gamma_2^{11} < x_v, x_v >$$
$$\vdots$$

Solve that system of equations to find the christoffeel symbols.
Theorem
The gaussian curvature ofa surface is invariatnt by local isometries.