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Differential Geometry
Notes
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Today

(a) Covariant derivative

(b) Parallel transport

(c) Geodesic

(d) G-B Theorem

Intrinsic geometry means we are looking at just surfaces. It lies in this coordinate.
w is a vector field on a surface. $w(u,v) = a(u,v)x_u + b(u,v)x_v$
Let's restrict w onto the curve $\alpha(t) = (u(t), v(t))$. Then $w(t) = a(\alpha(t))x_u + b(\alpha(t))x_v$ and

$$\begin{aligned} w'(t) &= a'(\alpha(t))\alpha'(t)x_u \\ &+ a(\alpha(t))(x_{uu}u'(t) + x_{vv}v'(t)) \\ &+ b'(\alpha(t))\alpha'(t)x_v \\ &+ b(\alpha(t))(x_{vu}u'(t) + x_{vv}v'(t)) \\ &= qx_u + rx_v + sN && \text{(for some } q, r, s \in R) \end{aligned}$$

If $Dw/dt = 0$ then that means the vector field is paralell to $\alpha'(t)$.
A vector field w along a parameterized curve $\alpha : I \to S$ is said to be parallel if $Dw/dt = 0$ for every $t \in I$.
Parallel transport in R^2 means all the vectors are parallel.
Proposition If w, v are parallel vector fields then $< w(t), v(t) >$ is constant.
Definition a nonconstant parameterized curve $\gamma : I \to S$ is said to be geodesic at $t \in I$ if the field of its tangent vectors $\gamma'(t)$ is parallel along γ at t. That is,

$$\frac{D\gamma'(t)}{dt} = 0$$

γ is a parameterized geodesic if it is geodesic for all $t \in I$.
Gauss-Bonnet

$$\int \int_S k \, ds = 2\pi\chi(S)$$

χ is the euler number for the surface. $\chi(S) = vertices - edges + faces$